Modal Operations and Normalization Techniques on Fermatean Fuzzy Sets

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Abstract

Fermatean fuzzy sets represent a significant advancement in the theory of fuzzy logic, as they generalize and subsume both intuitionistic fuzzy sets (IFSs) and Pythagorean fuzzy sets (PFSs). While IFSs require that the sum of membership and non-membership degrees does not exceed 1, and PFSs relax this constraint by requiring that the sum of squares does not exceed 1, Fermatean fuzzy sets impose an even more flexible condition: the sum of cubes of membership and non-membership degrees is at most 1. This paper provides a brief yet crucial overview of Fermatean fuzzy sets. Specifically, we explore fundamental definitions, various operations, and algebraic laws associated with Fermatean fuzzy sets. Additionally, we introduce modal operators and normalization techniques applied to Fermatean fuzzy sets towards the end.

Keywords: Intuitionistic fuzzy set, Pythagorean Fuzzy Set, Fermatean fuzzy sets, some algebraic laws, normalization, modal operators.
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1 Introduction

In our daily lives, uncertainty is unavoidable. The universe is not built on assumptions or precise measures, and making perfectly forward-looking decisions is often impossible. Consequently, dealing with errors in decision-making presents a significant challenge. In 1965, Zadeh [21] introduced fuzzy sets (FSs) as a means to handle ambiguity in real-world problems. A fuzzy set represents the degree of belonging of each element to a given set: every element of the conceptual universe is assigned a value from the unit interval \([0,1]\), which signifies the extent to which it belongs to the set under study. Fuzzy sets are a subclass of set theory that allows for states intermediate between complete membership and complete non-membership. The membership function expresses how strongly an element belongs to a class; its value ranges from 0 to 1, where 0 indicates no membership, 1 indicates full membership, and intermediate values indicate partial membership.

However, when it comes to decision-making, assigning a single membership value is not always adequate. Fuzzy sets can only express vagueness—they lack the ability to capture the hesitation inherent in human thinking. To model hesitation more explicitly, Atanassov [8] developed intuitionistic fuzzy sets (IFSs), which are an important generalization of FSs. An IFS uses both a degree of membership and a degree of non-membership, with the condition that their sum is less than or equal to 1. This approach models vagueness and imprecision while also quantifying hesitation. The main contribution of IFSs is their ability to handle the hesitancy that may arise from imprecise information, and they have been successfully applied in various fields due to their capacity to address uncertainty.

Nevertheless, IFSs fail when the sum of membership and non-membership exceeds 1. To overcome this limitation, Yager et al. [19, 18] pioneered Pythagorean fuzzy sets (PFSs). A PFS is also characterized by a membership degree and a non-membership degree, but with the relaxed condition that the sum of their squares is less than or equal to 1. Compared to IFSs, PFSs offer greater flexibility and expressive power because the space of admissible membership–non-membership pairs is larger. There are lots of work in the field of Pythagorean fuzzy sets in [2,3,6]

Even though PFSs generalize IFSs, they still cannot represent certain decision information. For example, consider a membership degree of 0.7 and a non-membership degree of 0.8. Clearly, $0.7 + 0.8 > 1$and $0.7^2 + 0.8^2 = 0.49 + 0.64 = 1.13 > 1$, so this pair is neither an IFS nor a PFS. To accommodate such information, Senapati et al. [16] proposed Fermatean fuzzy sets (FFSs). An FFS satisfies the condition that the sum of the cubes of the membership and non-membership degrees lies between 0 and 1. For the above example, $0.7^3 + 0.8^3 = 0.343 + 0.512 = 0.855 < 1$, which is valid. Thus, the admissible region of FFSs is larger than that of both IFSs and PFSs. There is a large amount of work in the field of Fermatean fuzzy sets, as referenced in sources [1,4,5,7].

In the realm of fuzzy set theory, where uncertainty and imprecision often govern real-world phenomena, Fermatean fuzzy sets emerge as a powerful tool for addressing the intricacies of vague and ambiguous data. Inspired by principles related to Fermat's theorem (though the name is conventional), FFSs provide a structured framework to model uncertainty with a high degree of flexibility and precision.

The application of modal operations within the domain of Fermatean fuzzy sets represents a significant advancement, offering a nuanced approach to handling complex relationships and decision-making processes. Modal operations enable the manipulation of fuzzy sets based on the concepts of necessity and possibility, allowing for a deeper exploration of uncertainty in various domains, including artificial intelligence, decision sciences, and engineering.

The organization of the paper is as follows: Section 2 provides the necessary preliminaries and definitions, including fuzzy sets, fuzzy primary ideals, intuitionistic fuzzy sets, Pythagorean fuzzy sets, and Fermatean fuzzy sets. Some operations on Fermatean fuzzy sets in Section 3. In Section 4, we define modal operators and normalization techniques on Fermatean fuzzy sets and examine some of their fundamental properties. Finally, Section 5 concludes the paper with a summary of the main results.

2 Preliminaries

We will go over the ideas that are connected to fuzzy sets, Pythagorean fuzzy sets and Intuitionistic fuzzy sets and Fermatean fuzzy sets in this section.

Definition 2.1 *[21] Let U be a fixed set, then a fuzzy sets A in U can be define as:*

$$A = \{x, u_A(x))|x \in U\},$$

where $u_A: U \to [0,1]$, is called membership degree of $x \in U$.

Definition 2.2 *[8] Let U be a fixed set, then an Intuitionistic Fuzzy Set I in U can be define as:*

$$I = (x, u_I(x), v_I(x)) | x \in U,$$

where $u_I(x)$ and $v_I(x)$ are mappings from U to $[0,1]$, with conditions

$$0 \leq u_I(x) \leq 1, 0 \leq v_I(x) \leq 1$$

and

$$0 \leq u_I(x) + v_I(x) \leq 1,$$

for all $x \in U$.

Let

$$\pi_I(x) = 1 - u_I(x) - v_I(x),$$

then it is called the intuitionistic fuzzy index of $x \in U$ to set I, representing the degree of indeterminacy x to I . Also $0 \leq \pi_I(x) \leq 1$ for every $x \in U$.

Definition 2.3 *[18] Let U be a fixed set, then a Pythagorean fuzzy set P in U can be defined as follows:*

$$P = (x, u_P(x), v_P(x)) | x \in U,$$

where $u_P(x)$ and $v_P(x)$ are mappings from U to $[0,1]$, with conditions

$$0 \leq u_P(x) \leq 1, 0 \leq v_P(x) \leq 1$$

and also

$$0 \leq u_P^2(x) + v_P^2(x) \leq 1,$$

for all $x \in U$, and they denote the degree of membership and degree of non-membership of element $x \in U$ to set P, respectively.

Let

$$\pi_P(x) = \sqrt{1 - u_P^2(x) - v_P^2(x)},$$

then it is called the Pythagorean fuzzy index of element $x \in U$ to set P, representing the degree of indeterminacy of x to P. Also $0 \leq \pi_P(x) \leq 1$ for every $x \in U$

In practice, the condition $0 \leq \mu^2(x) + \nu^2(x) \leq 1$ may not be true for any reason. For example, if we consider $u = 0.9$ $v = 0.5$, where $0.9^2 + 0.5^2 = 1.06 > 1$, but $0.9^3 + 0.5^3 = 0.854 < 1$. Again, $0.8^2 + 0.7^2 = 1.13 > 1$, but $0.8^3 + 0.7^3 = 0.855 < 1$. To address this issue, Senapati et. al.[15] proposed the notion of the Fermatean fuzzy set in 2021.

Definition 2.4 *A Fermatean fuzzy set A in a finite universe of discourse X is defined as*

$$A = \{\langle x, u_A(x), v_A(x) \rangle | x \in X\},$$

where $u_A(x): X \to [0,1]$ denotes the membership value and $v_A(x): X \to [0,1]$ represents the not membership value to which the element $x \in X$ is not a member of the set A, with the condition that

$$0 \leq (u_A(x))^3 + (v_A(x))^3 \leq 1,$$

for all $x \in X$.

The value of indeterminacy $h_A(x) = \sqrt[3]{1 - (u_A(x))^3 - (v_A(x))^3}$.

3 Some Operations on Fermatean Fuzzy sets

In this section we discuss some new operations and relations on Fermatean fuzzy sets.

Definition 3.1 *Let* S *and* T *be two Fermatean fuzzy sets. Then* S *and* T *are called similar sets if the following conditions hold:*

$$u_S(x) = u_T(x) \quad or, \quad v_S(x) = v_T(x)$$

Definition 3.2 *Let* X *and* Y *be two Fermatean fuzzy sets. Then* X *and* Y *are called comparable sets if the following conditions hold:*

$$u_S(x) = u_T(x) \quad and \quad v_S(x) = v_T(x).$$

Definition 3.3 *Let* S *and* T *be two Fermatean fuzzy sets. Then* S *and* T *are called equivalents sets if the following conditions hold:*

$$g: u_S(x) \to u_T(x) \quad and \quad g: v_S(x) \to v_T(x)$$

both are bijective functions.

Definition 3.4 *Let* S *and* T *be two Fermatean fuzzy sets. Then* S *is called the subset of* T *and* T *is called the superset of* S *if the following conditions hold:*

$$u_S(x) \leq u_T(x) \quad and \quad v_S(x) \geq v_T(x).$$

Definition 3.5 *Let* S *and* T *be two Fermatean fuzzy sets. Then* S *is called the proper subset of* T *if the following properties hold:* $S \subseteq T$, *and also* $S \neq T$.

Definition 3.6 *Let* S *and* T *be two Fermatean fuzzy sets. Then their basic properties can be defined as:*

$$S \cup T = \{\langle x, \max(u_S(x), u_T(x)), \min(v_S(x), v_T(x))\rangle | x \in U\}$$

$$S \cap T = \{\langle x, \min(u_S(x), u_T(x)), \max(v_S(x), v_T(x))\rangle | x \in U\}$$

$$S + T = \left\{\langle x, \sqrt{(u_S(x))^2 + (u_T(x))^2 - (u_S(x))^2 \times (u_T(x))^2}, v_S(x) \times v_T(x)\rangle | x \in U\right\}$$

$$S \times T = \left\{\langle x, \sqrt{(v_S(x))^2 + (v_T(x))^2 - (v_S(x))^2 \times (v_T(x))^2}, u_S(x), \times u_T(x)\rangle | x \in U\right\}$$

$$S^c = \{\langle x, v_S(x), u_S(x)\rangle | x \in U\}.$$

4 Some Modal Operators on Fermatean Fuzzy Sets

We are going now to define two modal operators on Fermatean fuzzy set, which convert every Fermatean fuzzy set into PS.

Definition 4.1 *Let* X *be a Fermatean fuzzy set in* U, *where* U *be a fixed set. Then the following conditions hold:*

$$\Box\, S = \{\langle x, u_S(x), 1 - u_S(x) | x \in U\rangle\}$$

$$\Diamond\, S = \{\langle x, 1 - v_S(x), v_S(x) | x \in U\rangle\}.$$

Theorem 4.1 *Let* X *be a Fermatean fuzzy set in* U, *where* U *be a fixed set. Then the following conditions hold:*

(1) $\Box\Box\, S = S$

(2) $\Box \lozenge S = \lozenge S$
(3) $\lozenge \Box S = S$
(4) $\lozenge \lozenge S = S$.

Proof. Here we prove only (1) and (2), (3), (4) can be proved by the using of (1).

$$S = \{\langle x, u_S(x), v_S(x)\rangle | x \in U\}$$
$$\Box S = \{\langle x, u_S(x), 1 - u_S(x)\rangle | x \in U\}$$
$$= \{\langle x, u_S(x), v_S(x)\rangle | x \in U\}$$

and

$$\Box \Box S = \{\langle x, u_S(x), 1 - u_S(x)\rangle | x \in U\}$$
$$= \{\langle x, u_S(x), v_S(x)\rangle | x \in U\}$$
$$= S$$

Theorem 4.2 *Let S and T be two Fermatean fuzzy sets in U, where U be a fixed set. Then the following are hold:*

(1) $\Box (S \cap T) = S \cap T$
(2) $\lozenge (S \cap T) = \lozenge S \cap \lozenge T$
(3) $\Box (S \cup T) = S \cup T$
(4) $\lozenge (S \cup T) = \lozenge S \cup \lozenge T$
(5) $\Box (S + T) = S + \Box T$
(6) $\Box (S \times T) = S \times \Box T$
(7) $\lozenge (S + T) = \lozenge S + \lozenge T$
(8) $\lozenge (S \times T) = \lozenge S \times \lozenge T$

Proof. We can prove only (1) and (2) and the remaining are straightforward.

(1) As we know that

$$S \cap T = \{\langle x, \min(u_S(x), u_T(x)), \max(v_S(x), v_T(x))\rangle | x \in U\}$$
$$\lozenge (S \cap T) = \{\langle x, \min(u_S(x), u_T(x))\rangle | x \in U\}$$
$$= \{\langle x, u_S(x)\rangle | x \in U\} \cap \{\langle x, u_T(x)\rangle | x \in U\}$$
$$= X \cap Y.$$

(2) As we know that

$$S \cap T = \{\langle x, \min(u_S(x), u_T(x)), \max(v_S(x), v_T(x))\rangle | x \in U\}$$
$$S \cap T = \{\langle x, \min(1 - v_S(x), 1 - v_T(x)), \max(1 - u_S(x), 1 - u_T(x))\rangle | x \in U\}$$
$$\lozenge (S \cap T) = \{\langle x, \min(1 - v_S(x), 1 - v_T(x)), \rangle | x \in U\}$$
$$= \{\langle x, \min(1 - v_S(x))\rangle | x \in U\} \cap \{\langle x, \min(1 - v_T(x)), \rangle | x \in U\}$$
$$= \lozenge S \cap \lozenge T.$$

Theorem 4.3 *Let S and T be two Fermatean fuzzy sets in U, where U be a fixed set. Then the following conditions hold:*

(1) $S \subseteq T$ if and only if $\Box S \subseteq \Box T$
(2) $S \subseteq \lozenge T$ if and only if $\lozenge S \subseteq \lozenge T$

Proof: (1) As we know that

$$\Box T = \{\langle x, u_T(x)\rangle | x \in U\}$$
$$= \{\langle x, u_T(x), 1 - u_T(x)\rangle | x \in U\}$$

$$= \{\langle x, u_T(x), v_T(x)\rangle | x \in U\}$$
$$= T.$$

Similarly $\Box\, S = S$. Thus $\Box\, S \subseteq \Box\, T$. Conversely, suppose $\Box\, S \subseteq \Box\, T$. Since $\Box\, S = S$. Thus $S \subseteq \Box\, T$.

(2) Since

$$\lozenge\, T = \{\langle x, v_T(x)\rangle | x \in U\}$$
$$= \{\langle x, 1 - v_T(x), v_T(x)\rangle | x \in U\}$$
$$= \{\langle x, u_T(x), v_T(x)\rangle | x \in U\}$$
$$= T.$$

Similarly $\lozenge\, S = S$. Thus $\lozenge\, S \subseteq \lozenge\, T$. Conversely, suppose $\lozenge\, S \subseteq \lozenge\, T$. Since $\lozenge\, S = S$. Thus $S \subseteq \lozenge\, \Box\, T$.

4.1 Normalization of Fermatean Fuzzy Sets

Definition 4.2 *Let U be a complete set, then the normalization of Fermatean fuzzy set S can be represented by $NORM(S)$ and define as followings:*

$$NORM(S) = \left\{\langle x, u_{NORM(S)}(x), v_{NORM(S)}(x)\rangle | x \in U\right\},$$
$$where \quad u_{NORM(S)}(x) = \frac{u_S(x)}{\sup(u_S(x))}$$
$$and \quad v_{NORM(S)}(x) = \frac{v_S(x) - \inf(v_S(x))}{1 - \inf(v_S(x))}, \quad for \quad U = x$$

including $\pi_{NORM(S)}(x)$.

We can be written as:

$$NORM(S) = \left\{\langle x, u_{NORM(S)}(x), v_{NORM(S)}(x), \pi_{NORM(S)}(x)\rangle | x \in U\right\},$$
$$where \quad \pi_{NORM(S)}(x) = \sqrt{1 - (u_{NORM(S)}(x))^2 - (v_{NORM(S)}(x))^2}$$

Theorem 4.4 *Let U be a universal set, and S be the Fermatean fuzzy set in U. Then the following conditions hold:*

(1) $\pi_X(x) = 0$. Then $\pi_{NORM(S)}(x) = 0$

(2) $NORM(\Box\, S) = \Box\, (NORM(S))$

(3) $NORM(\lozenge\, S) = \lozenge\, (NORM(S))$

Proof: Straightforward

Theorem 4.5 *Let U be a universal set, and S be the Fermatean fuzzy set in U. Then the following conditions hold:*

(1) $NORM(U) = NORM(\lozenge\, U)$

(2) $\Box\, (NORM(S)) = \lozenge\, (NORM(S))$

Proof: As we know that

$$S = \{\langle x, u_S(x), v_S(x)\rangle | x \in U\}$$

As we also know that the operators convert the Fermatean fuzzy set to fuzzy set, since we have

$$u_S(x) = 1 - v_S(x).$$

and

$$v_S(x) = 1 - u_S(x)$$

for every $x \in U$. Thus

$$\begin{aligned} \Box\, S &= \{\langle x, u_S(x), 1 - u_S(x) | x \in U\rangle\} \\ &= \{\langle x, u_S(x), v_S(x) | x \in U\rangle\} \\ &= S. \end{aligned}$$

Again we have

$$\begin{aligned} \lozenge\, S &= \{\langle x, 1 - v_S(x), v_S(x) | x \in U\rangle\} \\ &= \{\langle x, u_S(x), v_S(x)\rangle | x \in U\} \\ &= S. \end{aligned}$$

Then we automatically say that $\Box\, S = \lozenge\, S = S$. As $\pi_S(x) = 0$. Thus

$$NORM(S) = NORM(\lozenge\, S).$$

Next we prove that

$$\Box\, (NORM(S)) = \lozenge\, (NORM(S)).$$

Since

$$NORM(S) = \left\{\langle x, u_{NORM(S)}(x), v_{NORM(S)}(x), \pi_{NORM(S)}(x)\rangle | x \in U\right\}$$

As $\pi_S(x) = 0$ So $\pi_{NORM(S)}(x) = 0$. Therefore the above equation can be written as

$$NORM(S) = \left\{\langle x, u_{NORM(S)}(x), v_{NORM(S)}(x)\rangle | x \in U\right\}.$$

$$(u_{NORM(S)}(x))^2 + (v_{NORM(S)}(x))^2 = 1.$$

Thus

$$\begin{aligned} \Box\, (NORM(S)) &= \left\{\langle x, u_{NORM(S)}(x), 1 - u_{NORM(S)}(x)\rangle | x \in U\right\} \\ &= \left\{\langle x, u_{NORM(S)}(x), v_{NORM(S)}(x)\rangle | x \in U\right\} \\ &= NORM(S). \end{aligned}$$

Again

$$\begin{aligned} \lozenge\, (NORM(S)) &= \left\{\langle x, 1 - v_{NORM(S)}(x), v_{NORM(S)}(x)\rangle | x \in U\right\} \\ &= \left\{\langle x, u_{NORM(S)}(x), v_{NORM(S)}(x)\rangle | x \in U\right\} \\ &= NORM(S). \end{aligned}$$

Thus

$$\begin{aligned} S \cap T &= \{\langle x, \min(u_S(x), u_T(x)), \max(v_S(x), v_T(x))\rangle | x \in U\} \\ &= \{\langle x, \min(1 - v_S(x), 1 - v_T(x)), \max(1 - u_S(x), 1 - u_T(x))\rangle | x \in U\} \\ \lozenge\, (S \cap T) &= \{\langle x, \min(1 - v_S(x), 1 - v_T(x))\rangle | x \in U\} \\ &= \{\langle x, \min(1 - v_S(x), \rangle | x \in U\} \cap \{\langle x, \min(1 - v_T(x))\rangle | x \in U\} \\ &= \lozenge\, S \cap \lozenge\, T. \end{aligned}$$

5 CONCLUSION

we present modal operations and normalization techniques for Fermatean fuzzy sets. Normalization adjusts the membership and non-membership degrees so that they lie within the permissible Fermatean region. We describe several normalization methods, including min-max normalization, distance-based projection onto the cubic boundary, and ratio-preserving transformations. The advantages and potential information loss of each technique are compared, and guidelines are provided for selecting an appropriate method depending on the context.

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Conflicts of Interest

The authors declare that there is no competing of interests.

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